resulting expression is $(-1)^n\lambda^n[(n+1)a-\lambda]$, and is of the form demanded by our theorem by the case of $n+1$. The left-hand member of the resulting expression may be written so that its determinant is of order $n+1$,

$$\frac{na+a-\lambda}{na-\lambda}\cdot\begin{vmatrix} -\lambda, & 0, & 0, & \cdots, & 0 \\ a, & a-\lambda, & a, & \cdots, & a \\ a, & a, & a-\lambda, & \cdots, & a \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a, & a, & a, & \cdots, & a-\lambda \end{vmatrix} \qquad (1)$$

Here the $-\lambda$ of our multiplier appears as the element in the first row and first column of the determinant of order $n+1$. Add the second, third, and each of the following rows in turn to the first row obtaining an equal expression, the new determinant being exactly the same as that in (1) above except that the elements of the first row are now all $na-\lambda$. Multiply the factor preceding this determinant into the elements of the first row obtaining a third determinant which is again exactly like (1) except that the elements of the first row are now all $na+a-\lambda$. Subtract the second, third, and each of the following rows from the first, thus obtaining a determinant of order $n+1$ and of precisely the form demanded by our theorem if it is to be true in the case of $n+1$. Thus, if the relation is true for $n$, it is also true for $n+1$, and the induction is complete.

Also solved by A. L. Candy, P. J. da Cunha, L. D. Hand, A. M. Harding, R. A. Johnson, L. C. Mathewson, H. L. Olson, A. Pelletier, J. L. Riley, G. Y. Sosnow, Elijah Swift, and C. C. Yen.

**2744 [1919, 37]. Proposed by E. B. ESCOTT, Chicago, Ill.**

An insurance company computes its quarterly premiums by adding 6 per cent to the annual premium and dividing by 4. If a policyholder pays quarterly, what rate of interest is he paying?

## I. Solution by Elijah Swift, University of Vermont.

If we assume that this means that the policyholder sets aside the annual premium at the beginning of the year, pays the first of the quarterly premiums out of it, lets the remainder lie at interest for three months, then deducts the second premium, and so on, the interest will be compounded quarterly, and the present worth of the four premiums at the beginning of the year must equal the annual premium. If we call the annual premium, $4P$, and the (unknown) annual interest rate, $4i$, each quarterly premium will be $1.06P$ and we have the equation

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{(1+i)^2}+\frac{1.06P}{(1+i)^3}=4P.$$

This cubic may be solved by Horner's method, whence $4i = 16.11$ per cent.

If interest be compounded semi-annually, we have a quadratic,

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{1+2i}+\frac{1.06P}{(1+i)(1+2i)}=4P$$

whence $4i = 16.33$ per cent.

If interest be reckoned as simple (compounded annually) we have to solve the cubic

$$1.06P+\frac{1.06P}{1+i}+\frac{1.06P}{1+2i}+\frac{1.06P}{1+3i}=4P.$$

whence $4i = 16.54$ per cent.

In any case, then, the policyholder must pay over 16 per cent for the accommodation.

## II. Solution by the Proposer.

A (by algebra).

Let $P$ = annual premium, $p$ = quarterly premium $=\frac{1.06}{4}P$, $r$ = annual rate of interest.

$$P=\frac{400}{106}p.$$

We have the equation

$$(P - p)\left[1 + \frac{r}{4}\right]^3 - p\left[1 + \frac{r}{4}\right]^2 - p\left[1 + \frac{r}{4}\right] - p = 0.$$

Putting

$$1 + \frac{r}{4} = x$$

and substituting value of $P$, we have the equation

$$294x^3 - 106x^2 - 106x - 106 = 0.$$

Solving by Horner's Method, we have

$$x = 1 + \frac{r}{4} = 1.04028,$$

$$r = .16112 = 16.112 \text{ per cent compounded quarterly.}$$

B (by arithmetic). A more elementary and more "practical" method is the method by trial and error. A few trials will show that the rate is something over 16 per cent.

*First Trial.* Taking the rate as 16 per cent and the annual premium as 100, we have the scheme,—

| | |
|---|---|
| Annual premium due | 100.00 |
| First quarterly premium paid | 26.50 |
| | 73.50 |
| Interest for three months | 2.94 |
| | 76.44 |
| Second quarterly premium paid | 26.50 |
| | 49.94 |
| Interest for 3 months | 2.00 |
| | 51.94 |
| Third quarterly premium paid | 26.50 |
| | 25.44 |
| Interest for 3 months | 1.02 |
| | 26.46 |
| Fourth quarterly premium | 26.50 |
| First error | 0.04 |

We see that 16 per cent is slightly too small.

*Second trial.* Taking the rate as 16.2 per cent., we have, in the same way as before, an error of $+ .03$.

Forming a table

| Rate | Error |
|---|---|
| 16 | $-.04$ |
| 16.2 | $+.03$ |

By interpolation, the rate that will give zero error is

$$16 + \frac{4}{7} \times .2 = 16.114 \text{ per cent.}$$

If greater accuracy were required, repeat the computation with the last rate and interpolate again.

Also solved by G. N. Armstrong, H. N. Carleton, and H. L. Olson.

**2747 [1919, 72]. Proposed by DANIEL KRETH, Wellman, Iowa.**

In the right triangle $ABC$, right angle $C$, we have given on the hypotenuse the segments $AD = 15$, $DE = 10$, $EB = 15$, and the angle $DCE$ equal to the angle $ECB$. Find the angle $DCE$, and the sides $AC$ and $BC$.